Asymptotic Behavior of In-Band Crosstalk Noise in WDM Networks

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***Abstract*—In-band crosstalk noise can pose important limitations in an optical network. To calculate the bit error rate (BER), the crosstalk–crosstalk beating noise is usually ignored in the literature. In this letter, it is shown how the crosstalk–crosstalk noise can be taken into account in the analysis of an optical receiver in the case of many independent interferers. It is shown that the crosstalk–crosstalk noise can influence the value of the BER, change the optimum receiver threshold, and introduce some power penalty.**

***Index Terms*—Crosstalk, error analysis, optical receivers, wavelength-division multiplexing (WDM).**

I. Introduction

THE PERFORMANCE of wavelength-division-multiplexing (WDM) networks can be severely limited by the presence of in-band optical crosstalk noise [1]. In-band crosstalk noise arises at optical cross connects because, due to their imperfect characteristics, a small delayed version of the signal or a small portion of light from other channels at the same wavelength (in a network with wavelength reuse) is routed along the same path as the signal. Since in-band crosstalk noise is at the same wavelength as the signal, it cannot be removed using additional filtering and can degrade the bit error rate (BER) at the receiver.

If the receiving photodiode is assumed to act as a square-law device, then there are two in-band crosstalk noise contributions present at the receiver: one resulting from the beating of the optical signal with the optical crosstalk noise, and one from the beating of the optical crosstalk noise with itself. The crosstalk–crosstalk noise is usually neglected in the literature [1]. In this letter, the effect of the crosstalk–crosstalk beating noise is considered, in the limit of many independent interfering channels. It is shown that even in the presence of optical amplifier noise, which is often considered as the major noise contribution in an optical network, the crosstalk–crosstalk noise can influence the value of the BER, change the optimum receiver threshold, and introduce some power penalty.

II. Mathematical Considerations

The moment generating function (MGF) $M_Z(s)$ of the decision variable Z (in photoelectrons) at the receiver is defined as the expected value of e^{sZ}. If $M_Z(s)$ is known, then the calculation of the BER can be accomplished using the saddle point method [2, Ch. 5]. In this letter, an optically preamplified receiver with an electrical integrate-and-dump filter and an ideal rectangular optical filter will be considered. The quantum efficiency of the photodetector will be assumed equal to unity. Under these assumptions the MGF $M_{Z|X}(s)$ of Z conditioned on the energy X (in photons) of the incident optical field at the input of the amplifier, is given by [2, Ch. 7]

$$M_{Z|X}(s) = \left(\frac{1}{1-N_0 s}\right)^L \exp\left(\frac{XGs}{1-N_0 s}\right). \tag{1}$$

In (1), $N_0 = n_{\mathrm{sp}}(G-1)$ is the power spectral density of the amplified spontaneous emission (ASE) noise, while G and n_{sp} are the gain and the spontaneous emission parameter of the optical amplifier respectively. $L+1$ is equal to the product BT of the bandwidth B of the optical filter and the bit duration T (L is assumed to be an integer). The MGF $M_Z(s)$ of Z unconditionally of X, can be calculated by taking the expected value of $M_{Z|X}(s)$ with respect to X. The optical field at the input of the amplifier can be represented in complex notation as $x(t) = \mathrm{Re}\{A(t)\exp(j2\pi f_0 t)\}$ where $A(t)$ is the envelope of the optical field, and f_0 the optical frequency. If the desired signal and the in-band crosstalk interfering components have the same pulse variation $g(t)$, then $A(t)$ is given by

$$A(t) = \sum_{m=0}^{M} c_m g(t) e^{j\phi_m}. \tag{2}$$

In (2), c_m are the amplitudes of the signal ($m=0$) and of the crosstalk components ($m>0$), ϕ_m is the phase difference between the signal and interferer $m>0$ ($\phi_0 = 0$), and M is the number of interferers. The ϕ_m are caused by the phase noise of the LASER sources and are assumed uniformly distributed in [0, 2π]. The energy of the signal is given by

$$\begin{aligned} X = \tfrac{1}{2}\int_0^T |A(t)|^2\,dt &= \sum_{k=0}^{M}\sum_{n=0}^{M} c_k c_n e^{j(\phi_k-\phi_n)} \\ &= c_0^2 + 2c_0\sum_{k=1}^{M} c_k\cos\phi_k + \sum_{k=1}^{M}\sum_{n=1}^{M} c_k c_n e^{j(\phi_k-\phi_n)}. \end{aligned} \tag{3}$$

In (3) the factor 1/2 in front of the integral of $|A(t)|^2$ is due to the complex notation adopted for $x(t)$ [2, Ch. 7]. It is also assumed that $g(t)$ is normalized so that $1/2\int_T |g(t)|^2\,dt = 1$. Due to this normalization, c_0^2 is the number of photons in the signal while c_m^2 is the number of photons of interferer $m>0$. As seen by (3), the decision variable can be decomposed in three parts: the signal-signal beating (c_0^2), the crosstalk-signal beating [first sum of the right-hand side of (3)], and the crosstalk–crosstalk

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beating (second sum). Taking the expected value of (1) with respect to X, we obtain

$$M_Z(s)=E\left\{M_{Z|X}(s)\right\}=\left(\frac{1}{1-N_0 s}\right)^L M_X\left(\frac{Gs}{1-N_0 s}\right) \quad (4)$$

where $M_X(s)=E\{e^{sX}\}$ is the MGF of X and $E\{.\}$ denotes expected value. If the crosstalk–crosstalk contribution in (3) is ignored, then due to the central limit theorem [3, Ch.1], as the number of interferers M becomes large, X becomes Gaussian with mean value equal to c_0^2, while its standard deviation σ_g^2 and MGF $M_X(s)$ are given by

$$\sigma_g^2=2c_0^2\sum_{m\geq 1} c_m^2 \quad (5)$$

$$M_X(s)=\exp\left(c_0^2 s+\sigma_g^2 s^2/2\right). \quad (6)$$

Using (6) and (4), it is possible to calculate the MGF of the decision variable when the crosstalk–crosstalk noise influence is ignored. To calculate the MGF of X when the crosstalk–crosstalk noise is taken into account, let R and V be given by

$$R=\sum_{m\geq 1} c_m\cos\phi_m, \quad V=\sum_{m\geq 1} c_m\sin\phi_m. \quad (7)$$

The mean value of R and V is $E\{R\}=E\{V\}=0$. When $M\rightarrow\infty$, the random variables R and V asymptotically become Gaussian and their joint statistical behavior is completely determined by their covariance $\rho=E\{RV\}$. If $\rho=0$, then R and V are mutually independent [3, Ch. 1]. Using (7), ρ is written as

$$\rho=E\{RV\}=\sum_{m\geq 1}\sum_{k\geq 1} c_m c_k E\{\cos\phi_m\sin\phi_k\}=0. \quad (8)$$

Equation (9) holds because, for uniformly distributed ϕ_l inside [0, 2π], one has $E\{\cos\phi_m\sin\phi_k\} = E\{\cos\phi_m\}$ $E\{\sin\phi_k\} = 0$ for $m\neq k$ and $E\{\cos\phi_m\sin\phi_m\} = 1/2E\{\sin(2\phi_m)\} = 0$. Since $\rho=0$, the Gaussian random variables R and V are independent.

Using (7) and (3), X is expressed as $X=(c_0+R)^2+V^2$, i.e., as the sum of the squares of the independent Gaussian random variables c_0+R and V. Therefore, X has a noncentral chi-square distribution [3, Ch. 1] with probability density function (pdf) $f_X(x)$ and MGF $M_X(s)$ given by

$$f_X(x)=\frac{1}{\sigma^2}\exp\left(-\frac{x+c_0^2}{\sigma^2}\right)I_0\left(\frac{2c_0\sqrt{x}}{\sigma^2}\right) \quad (9)$$

$$M_X(s)=\frac{1}{1-\sigma^2 s}\exp\left(\frac{c_0^2 s}{1-\sigma^2 s}\right) \quad (10)$$

where σ^2 is the number of photons of the crosstalk noise

$$\sigma^2=E\{R^2\}+E\{V^2\}=\sum_{m=1}^{M} c_m^2. \quad (11)$$

The ratio c_0^2/σ^2 represents the signal-to-crosstalk ratio in the optical domain. It is sometimes convenient to normalize the optical field so that $\sigma^2=1$. By applying the saddle point method, $M_Z(s)$, which is determined by (10) and (4), can be used to estimate the BER, when the crosstalk–crosstalk noise is included.

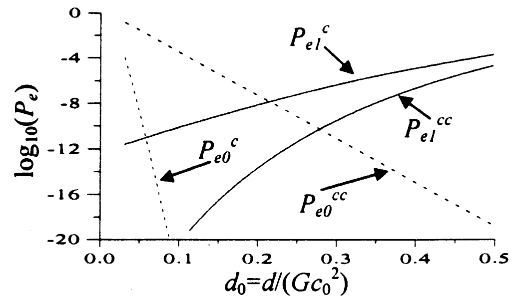

Fig. 1. Relation between the receiver threshold and the error probabilities for bit $b_s=1$ (solid lines) and $b_s=0$ (dashed lines) and the threshold at the receiver d normalized with respect to Gc_0^2.

III. Importance of the Crosstalk-Crosstalk Noise

The MGFs calculated in the preceding section will now be used to estimate the importance of the crosstalk–crosstalk noise in the performance of the system, using the saddle point method [2, Ch. 5]. In the case where the signal bit is $b_s=1$, the MGF can be obtained using (4) and either (6) or (10) depending on whether the crosstalk–crosstalk noise is included or not, respectively. Assuming a simple nonreturn-to-zero (NRZ) ON–OFF keying, c_0^2 will be given by $c_0^2=P_{\text{in}}T/(hf_0)$ where P_{in} is the incident optical power of the signal at the amplifier input and h is Planck's constant. In the case where $b_s=0$, assuming a perfect extinction ratio, the MGF is obtained by the same procedure, setting $c_0=0$. In order to take into account the electrical thermal noise, the MGF of the decision variable is multiplied by $\exp(\sigma_{\text{th}}^2 s^2/2)$ which is the MGF of the thermal noise. The power of the thermal noise is equal to $\sigma_{\text{th}}^2=2k_B T_K T/(q^2 R_L)$, where k_B is Boltzmann's constant, R_L the load resistor of the photodetector, T_K the temperature (in kelvin), while q is the charge of the electron.

In Fig. 1, the error probabilities $P_{e,i}^{cc}$, when the crosstalk–crosstalk noise is included and the signal bit is $b_s=i$, are plotted with solid lines for various values of the decision threshold d at the receiver (normalized with respect to the number of photoelectrons Gc_0^2 of the signal at the photodiode output) assuming that $P_{\text{in}}=-30$ dBm, $G=30$ dB, $n_{\text{sp}}=1$, $T=100$ ps (corresponding to a bit rate of 10 Gb/s), $B=10/T=100$ GHz, $R_L=100\ \Omega$, and $c_0^2/\sigma^2=100$ (implying an optical signal-to-crosstalk ratio of 20 dB). Also plotted with dashed lines are the error probabilities $P_{e,i}^{c}$ when the crosstalk–crosstalk noise is neglected. The error probabilities predicted by the two models are quite different when $b_s=0$ since, in this case the signal–crosstalk noise is not present and the crosstalk–crosstalk beating noise becomes a major noise contribution. The difference between the two models is significant but not as pronounced in the case where $b_s=1$, since both models take into account the signal-crosstalk beating noise which is larger than the crosstalk–crosstalk noise. It is interesting to note that the error probability when $b_s=1$ is less when the crosstalk–crosstalk term is included. This result can be justified by considering the special case of no optical amplification ($G=1$, $N_0=0$) and no thermal noise.

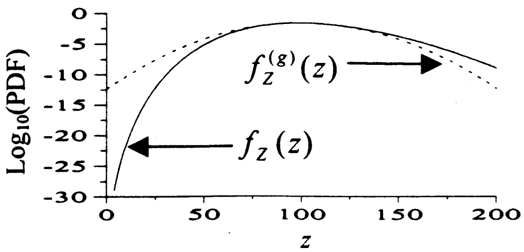

Fig. 2. Pdf of the decision variable Z when the crosstalk–crosstalk influence is ignored (dashed lines) and when it is taken into account (solid lines) for $c_0 = 10$ and $\sigma^2 = 1$ without ASE and thermal noise.

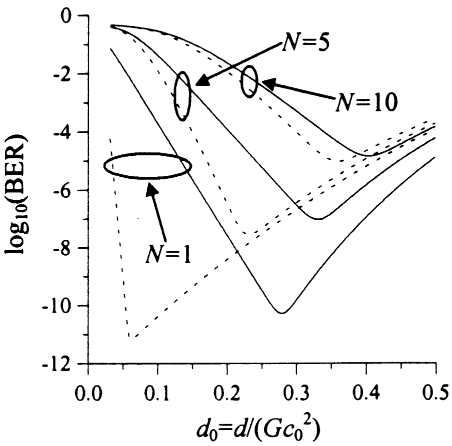

Fig. 3. Calculated values of the BER at the receiver, when the crosstalk–crosstalk noise is taken into account (solid lines) and when it is neglected (dashed lines) assuming that the signal and the crosstalk components pass through a cascade of $N = 1$, 5, or 10 amplifiers before reaching the photodiode.

In this case, using (4), it is deduced that $M_Z(s) = M_X(s)$ and hence the pdf $f_Z(z)$ is the same as that of X. When the crosstalk–crosstalk is taken into account, the pdf of Z is given by (9) and is plotted with a solid line in Fig. 2, for $\sigma^2 = 1$ and $c_0 = 10$. As stated in Section II, if the crosstalk–crosstalk noise is neglected, the pdf $f_Z^{(g)}(z)$ of Z is Gaussian with mean value equal to c_0^2 and standard deviation given by (5). The pdf $f_Z^{(g)}(z)$ is also plotted with dashed lines in Fig. 2. It is evident that the left tails of the Gaussian pdf $f_Z^{(g)}$ are above those of f_Z. Since the error probability in the case $b_s = 1$, is determined by the left tails of the pdf of the decision variable [2, Ch. 5], it is evident that inclusion of the crosstalk–crosstalk noise leads to a smaller error probability.

The value of the BER of the system obtained as the average of the error probabilities obtained in the cases $b_s = 1$ and $b_s = 0$ are plotted in Fig. 3. In the figure, N denotes the number of optical amplifiers that both the signal and the crosstalk components pass before reaching the photodiode. The last amplifier is that of the preamplified receiver and, in the case $N = 1$, only this amplifier is assumed. The rest of the system parameters are those of Fig. 1. For $N = 1$, the optimum threshold predicted by the two models is quite different and the minimum BER also differs in about one order of magnitude. However, as N begins to increase, the ASE noise accumulates and the crosstalk–crosstalk noise becomes less important reducing the difference between the two models.

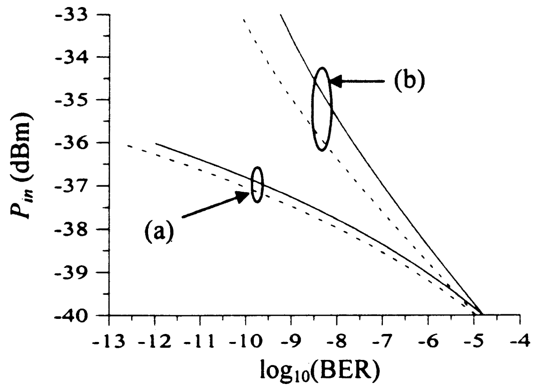

Fig. 4. Relation between the BER and the power P_{in} incident at the amplifier input, (a) when the power of the crosstalk components does not change, (b) when the signal-to-crosstalk ratio remains constant. The solid lines represent the BERs calculated when the crosstalk–crosstalk noise is included and the dashed lines the BERs when it is not.

The decrease of the BER experienced by increasing the incident power P_{in} of the signal is depicted in Fig. 4, for $c_0^2/\sigma^2 = 100$ at $P_{\text{in}} = -40$ dBm and $N = 1$, $n_{\text{sp}} = 1$, $G = 30$ dB, $T = 100$ ps, $R_L = 100\ \Omega$, $B = 100$ GHz. Two cases are examined: a) the incident power is increased and the crosstalk power remains constant; and b) the incident power is increased and the signal–crosstalk ratio remains constant. The latter case is considered because in many networks, such as an arrayed-waveguide grating interconnection, increasing the power of the transmitting channels results in an increase of the power of the crosstalk noise by the same amount [4]. By comparing the values of P_{in} required to achieve a BER equal to 10^{-9}, it is deduced that the inclusion of the crosstalk–crosstalk noise causes a power penalty of 0.2 dB in Case a, and 1.5 dB in Case b. Depending on the designer's crosstalk power penalty requirement, it is, therefore, deduced that crosstalk–crosstalk noise can become an important issue in system design.

IV. Conclusion

In this letter, the influence of the in-band crosstalk–crosstalk beating noise in the performance of a preamplified receiver was investigated in the limit of a large number of interferers. The results obtained suggest that the crosstalk–crosstalk beating noise cannot be neglected in general and can influence the BER of the system and the choice of the optimum threshold.

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