Game-theoretic Analysis of Competition between Access Service Providers utilizing a Nash Genetic Algorithm

Abstract

Fiber-to-the-home (FTTH) technology is a promising solution for providing advanced service delivery to end-users, but its implementation requires substantial capital expenditures. To minimize investment risks and aid decision-making for access service providers, we propose a game theoretic framework based on a modified Nash genetic algorithm. We illustrate how this framework can be applied to analyze the competition between access providers offering a flat-rate FTTH service on multiple geographical areas. Each provider determines its price for all areas simultaneously and decides whether to invest on a particular area depending on the anticipated revenues. Two distinct demand models are adopted to describe different types of consumer behavior. A solution engine, based on a modified mixed-variable Nash genetic algorithm is implemented under an open-source license. The significance and practical implications of the equilibrium points obtained for both single and multiple area games are discussed. The proposed framework and the solution engine developed, aid both providers and regulatory bodies to analyze competitive environments. They can also be used to implement decision support tools for similar problems as well. The paper concludes by pointing out further research directions in this context.

Keywords: Game theory, Nash genetic algorithm, access providers, communications, competition, broadband services

1 Introduction

The communications industry plays a fundamental role in the global economy and has been driving the development of the internet and the world wide web for decades. The market for communication services is continuously growing and was valued at 1.7 trillion USD in 2019 with at an annual growth rate of 5% until 2027 [1]. The COVID-19 pandemic accelerated the adoption of e-learning and teleworking, highlighting the need for upgrades in information and communication (ICT) infrastructures [2]. The advent of 5G [3] ushered in new business opportunities in domains such as massive machine-type, enhanced mobile broadband and ultra-reliable low latency communications [4]. Access networks providing connectivity to user premises are crucial to meeting the performance requirements of 5G.

Various actors are involved in the development of the communications industry, including users, device manufacturers, network, content, service, cloud providers and regulatory bodies [5]. Given the high stakes, investment decisions need to be based on rigorous frameworks. The motivation behind this work, lies in the establishment of a strategic analysis tool for access provider competition operating at a national or regional scale. We focus on Fiber-to-the-Home (FTTH) which is considered the next generation evolution of access networks. Due to their inherent low propagation loss, optical cables provide unprecedented bandwidth $\times$ distance products [6], ensuring reliable gigabit-per-second access rates at large distances, unlike wireless technologies where issues such as line-of-sight, terrain, etc may undermine service delivery. FTTH is nevertheless associated with high deployment costs involving roadworks, cable installation, termination, testing and investments in active equipment. This renders its deployment an economic rather than technical issue, determined by factors such as customer density, average income per capita, existing copper infrastructure, subsidization policies etc. Providers tend to deploy their FTTH network, starting from densely populated areas and gradually expanding to sparsely populated ones, taking competition into account. On the other hand, regulatory bodies need to analyze such strategies to regulate the market, taking the user's best interest at heart [7]. In our work, we address this problem in the context of *game theory*, which is an established mathematical framework for analyzing the interaction among rational competing agents [8].

Game theory offers valuable insights in scenarios featuring multiple rational decision-makers, each striving to make their own locally optimal decisions. Nash equilibria serve as solutions that establish stable points where all parties simultaneously achieve their optimization goals. These equilibria represent situations in which rational players lack any motivation to alter their decisions. Given that access providers generally act as rational players seeking to maximize their revenues in a competitive landscape, game theory emerges as an apt framework for analysis in these contexts [9]. Finding Nash equilibria presents a considerable challenge, especially when dealing with complex utility functions and large decision spaces. Analytical solutions are often only attainable under simplified assumptions. In cases involving the intricate interplay of strategic decision-making, numerical methods become a necessity. One such method is the application of a Nash genetic algorithm [10]. This approach combines the fundamental principles of genetic algorithms and game theory. Players are allocated chromosome populations reflecting different potential decisions. During each iteration of the algorithm, players in turn strive to locally optimize their utility functions. Over successive iterations, the algorithm steadily converges towards a solution that corresponds to a Nash equilibrium. This iterative and evolutionary process enables the algorithm to navigate a landscape of strategic interactions and identify equilibrium points efficiently.

Using this framework, the paper addresses a critical issue involving the competition among multiple FTTH providers vying for market dominance in various geographic areas. These regions may exhibit diversity in terms of population, service demand, and deployment costs. Each provider must make strategic decisions regarding the expansion of services into specific areas and the adjustment of a uniform service fee, all with the goal of optimizing individual profits. The primary objective is to pinpoint Nash equilibrium points where providers lack any incentive to modify their tariff policies and expansion decisions and market has reached a stable state. Adapting this framework to the situations at hand, providers can evaluate the best course of action while regulatory bodies can shape their policy decisions in order to achieve the desired outcome in terms of broadband service penetration, etc.

The rest of the paper is organized as follows: In Section 2, we present existing literature in game theory approaches and explain our envisaged contribution compared to it. In Section 3, we further describe the two alternative demand models. In Section 4, we highlight the details of the genetic algorithm used in order to calculate the Nash equilibrium. Section 5, provides some indicative results under different settings interpreted from a strategic point-of-view. Section 6 summarizes our work and provides some directions for future work.

2 Related work an contribution

Game theoretic approaches are frequently used in operations research [11–15] and have been applied in various aspects of ICT, not necessarily focusing on competition. A detailed literature review [9, 16, 17] is outside the scope of this paper and we only mention some pertinent works. In [18], a slotted resource allocation game with several providers is analyed. Unlike our case, this work considers wireless service providers having fixed capacity during each time slot and user demand can be split among providers. In [19], both the effect of pricing and quality-of-service (QoS) decisions of service providers is considered using game theory. The authors assume a single area game, with a linear demand model and no option is given to the providers not to expand. In [20], real-options and game theory are applied to evaluate ICT investments and oligopoly under multi-criteria perspectives. The paper considers two firms that can decide whether to invest and how much to produce under a utility function inspired by the analytic hierarchy process (AHP). The framework is applied in the case of a construction company planning of laying fiber cables along a newly constructed motorway and offering dark-fiber service to other telecom providers. This situation is quite different than our scenario considered, where the service providers compete over multiple areas under various user demand models. In [21] game theory is used to study the collusion and competition strategies between service providers. The authors adopt evolutionary and

Hotelling approaches to analyze methods of eliminating operator collusion at a government level. In [22], a general game-theoretic treatment of the oligopoly among Slovakian service providers is given. In [23], the impact of bundling in network provider strategy is analyzed, using evolutionary game theory. The study focuses on the problem of the increasing adoption of over-the-top services and the threat it poses to telecommunication providers.

Compared to existing literature, we adopt the game-theoretic approach to describe the area-by-area competition among providers in FTTH deployment. In our scenario, each provider j can decide whether or not to expand in a specific area i and can also set the price p_j it charges (considered flat in all regions). In practice, regions may correspond to provinces, municipalities or communities, each with its own deployment-related costs [24]. In this context, Nash equilibria determine the optimal expansion and pricing strategy in the sense that no competitor has incentive to deviate from it. A number of other frameworks have been considered in ICT such as auction mechanisms especially for cloud resource management and pricing [25] or network bandwidth allocation [26, 27]. The present work focuses on provider competition described by alternative demand models discussed below. We do not focus on Stackelberg games [9, 28] where one or more providers commit to expanding and the rest adapt to this action. In FTTH, new cables must be installed at the customer premises anyway, unlike previous generation copper-based access technologies where decisions of the incumbent operator could largely determine the market, due to its existing infrastructure. Given the demographics of each area i and the prices $\{p_k\}$ charged by the providers, the number of area subscribers n_{ij} attracted from access provider j is estimated by the demand model. Examples assume n_{ij} being a linear superposition of p_k [29], attraction models [30], finite-state continuous-time Markov chains [31], approaches based on the standard microeconomic framework, [32], simulation [33] and empirical approximations based on curve fitting [34]. Except but special cases, the demand model intricacies may render analytical solution for Nash equilibria intractable and one must resort to numerical techniques.

2.1 Contribution

In our work, we modify a Nash genetic algorithm (Nash GA) based on the work of [10] to provide a unified approach for mapping both real and binary decision variables using a scheme proposed for conventional GAs [35]. Compared to previous works, our contribution can be summarized as follows:

- We focus on the case of service providers competing over the provision of FTTH service in a region. We consider both single and multi-area games where providers have the option to expand or not in certain areas depending on the anticipated revenues. This scenario has not been previously analyzed in the literature with this level of detail.

- We highlight the details of a Nash GA suited for access provider competition, including initialization, convergence and unified treatment of decision variables. We also elaborate on the practical implications of the obtained Nash equilibria. Although Nash genetic algorithms have been used recently for analyzing general competitive games [36], here we formulate a game which is particularly suited for the communications sector and the scenario at hand.
- We consider two types of consumer behavior. The *limited interaction* case is based on an attraction model [37] where the relative proportion of subscribers attracted by any two providers deciding to expand depends solely on the prices they charge. Under this model, the interaction between providers is therefore limited. We also explore an *enhanced interaction* model, where customers make informed decisions based on the cheapest price offered in their area. It is this price that ultimately determines the market size and competing providers attracts customers portions depending on the difference of the prices and the minimum price.
- In addition, we make our `Python` implementation freely available on the web [38] under an open-source license. This enables researchers to adapt our framework to their particular case studies. To our knowledge, no such open-source implementation currently exists.

Our proposed framework can be extended to incorporate alternative demand modelling. It is particularly useful when the analytic solution for Nash equilibria can not be found (e.g. due to demand-related model complexities). In all cases assumed here, the algorithm converges well and numerically locates an equilibrium point even in multiple area games, where the dimension of the search space increases.

3 Provider interaction modeling

In this section, we consider the main components needed in order to describe the provider interaction from a game-theoretic standpoint.

3.1 Game-theoretic framework

The *action profile* $\mathbf{s}_j$ of each provider j consists of the variables that it can vary to obtain the optimal profitability. These are:

- the price p_j charged for the service, assumed flat for all areas i.
- the variables b_{ij} reflecting whether or not he invests on i ($b_{ij} = 1$ and $b_{ij} = 0$ respectively).

We therefore can write:

$$\mathbf{s}_j = \{p_j, b_{1j}, \cdots, b_{Nj}\} \tag{1}$$

where N is the number of areas considered. Note that p_j are continuous variables while b_{ij} are binary. The decisions of provider j are prioritized under a *utility function*, u_j, related to the actions of other players as well. The utility function $u_j = u_j(\mathbf{s})$ maps the action profile $\mathbf{s}$ which comprises of the ensemble of combined player actions,

$$\mathbf{s} = (\mathbf{s}_1, \cdots, \mathbf{s}_j, \cdots, \mathbf{s}_M) \tag{2}$$

to real numbers. In (2), M is the number of providers. Let $\mathbf{s}_{-j}$ be the action profile obtained without considering the action of player j,

$$\mathbf{s}_{-j} = (\mathbf{s}_1, \cdots, \mathbf{s}_{j-1}, \mathbf{s}_{j+1}, \cdots, \mathbf{s}_M) \tag{3}$$

and $(\mathbf{s}_j^*, \mathbf{s}_{-j})$ be the action profile obtained from $\mathbf{s}$ when the action of player j is replaced by $\mathbf{s}_j^*$,

$$(\mathbf{s}_j^*, \mathbf{s}_{-j}) = (\mathbf{s}_1, \cdots, \mathbf{s}_{j-1}, \mathbf{s}_j^*, \mathbf{s}_{j+1}, \cdots, \mathbf{s}_M) \tag{4}$$

Nash's equilibrium corresponds to an action profile $\mathbf{s}^* = (\mathbf{s}_1^*, \cdots, \mathbf{s}_M^*)$ where no player can improve its utility function by unilaterally changing his action, i.e.:

$$u_j(\mathbf{s}_j^*, \mathbf{s}_{-j}^*) \geq u_j(\mathbf{s}_j, \mathbf{s}_{-j}^*) \tag{5}$$

for every player j and every possible action $\mathbf{s}_j$ of player j. This situation corresponds to a *steady state* $\mathbf{s}^*$ that when reached, the players do not have any reason to choose different actions [8]. We assume that players know each others' preferences, and behave rationally in a predictable way, which is typical for network providers seeking to maximize their profits. The utility function of each provider is related to its profit as discussed in the next section.

3.2 Utility functions

The utility function reflects the total profit of each provider from all areas in the game. Fig. 1 shows the main parameters involved. These are:

- the number n_{ij} of customers attracted by each provider j in each area i.
- a fixed cost component d_{ij} related to deploying the service by provider j in area i, including investments in the provider's points-of-presence (PoPs), etc.
- the average cost c_{ij} of connecting each subscriber including roadworks, fiber cable installation and optical equipment.
- the variables b_{ij} which denote whether provider j decides to expand in area i.
- the total cost C_{ij} incurred if provider j decides to move in area i. As explained in Figure 1, this is given by:

$$C_{ij} = c_{ij} n_{ij} + d_{ij} \tag{6}$$

- the fee p_j charged by each provider j in all areas he decided to expand.

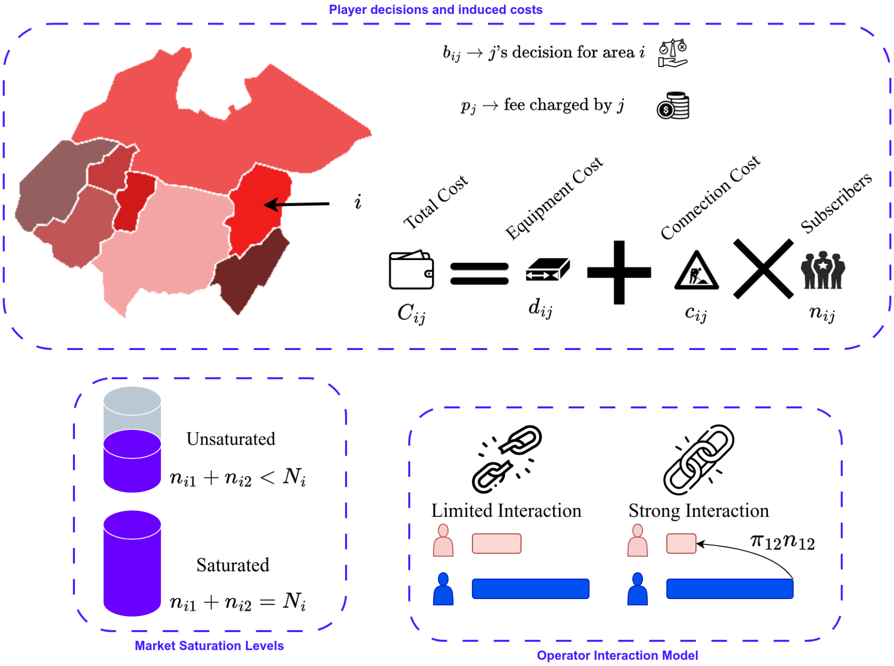

Fig. 1: Illustration of the various parameters involved in the utility function calculation, the market saturation levels and the provider interaction models.

Both d_{ij} and c_{ij} may depend on network architecture. In passive optical network (PON) deployments, fiber is shared from the PoP of the provider up to an optical splitter located in a central point of a neighborhood. Customers are then connected to the splitter by dedicated fibers. This approach leads to reduced fiber roll-out and is generally preferred [6]. In any case, the profit P_{ij} obtained for provider j in area i is $P_{ij} = p_j n_{ij} - C_{ij}$ if $b_{ij} = 1$ or zero otherwise. Equivalently:

$$P_{ij} = (p_j n_{ij} - C_{ij}) b_{ij} \tag{7}$$

The utility function $u_j(\mathbf{s})$ is obtained by summing up profits over all areas i:

$$u_j(\mathbf{s}) = \sum_{i=1}^{N} P_{ij} = \sum_{i=1}^{N} (p_j n_{ij} - C_{ij}) b_{ij} \tag{8}$$

We relate n_{ij} in (8) to $\mathbf{s}$ given by (2) through models discussed in the next subsection. The total number of subscribers attracted by each provider is simply:

$$n_j = \sum_i n_{ij} \tag{9}$$

3.3 Demand modeling

Fig. 1 illustrates two market regimes: the unsaturated regime, where there are some customers that have not yet acquired the service and the saturated regime where all customers have subscribed to one of the providers. We also illustrate two different interaction models: the limited interaction model where there is little interaction between providers and an enhanced interaction model. The details will be explained in the next subsections.

3.3.1 Limited interaction model

The *limited interaction model* is based on existing attraction models widely used in the literature [37] and more specifically multi-nomial logit (MNL) models [39, 40]. The number of customers attracted varies exponentially with price,

$$n_{ij} \propto b_{ij}\mathrm{e}^{-\alpha p_j} \tag{10}$$

In (10), the constant α captures the ease with customers are attracted to providers. If j chooses not to expand in region i ($b_{ij} = 0$) then no customers are attracted.

Fig 1 illustrates the situation when $M = 2$ providers are competing in one area i. Each attracts a number of subscribers n_{ij} from the total potential customers N_i. Naturally, the provider with the lowest price attracts more customers. If the prices are high, there can be many customers that decide not to adopt, i.e. $n_{i1} + n_{i2} < N_i$, still relying on previous generation copper-based last mile technologies. In this *unsaturated* market regime, we have:

$$n_{ij} = b_{ij}N_i\mathrm{e}^{-\alpha p_j} = N_i e_{ij} \tag{11}$$

where e_{ij} is given by:

$$e_{ij} = b_{ij}\mathrm{e}^{-\alpha p_j} \tag{12}$$

and corresponds to the market share of each provider. Note that the ratio of customers attracted by any two providers $j = q$ and $j = r$ deciding to invest ($b_{iq} = b_{ir} = 1$), is:

$$\frac{n_{iq}}{n_{ir}} = \frac{e_{iq}}{e_{ir}} = \mathrm{e}^{-a(p_q - p_r)} \tag{13}$$

The above equation implies that the ratio of attracted customers between any two providers depends on their price difference. Using (11), it is easy to show that if a provider invests ($b_{ij} = 1$), then the normalized rate of change r_j of attracted customers with price, equals:

$$r_j \overset{\text{def}}{=} \frac{1}{n_{ij}}\frac{\partial n_{ij}}{\partial p_j} = \alpha, \text{ assuming that } b_{ij} = 1 \tag{14}$$

Equation (14) suggests that the relative rate of change of attracted customers with respect to price of an investing provider is equal to α assuming unsaturated market. Let E_i be defined as:

$$E_i = \sum_j e_{ij} \tag{15}$$

In the unsaturated regime, $E_i < 1$ and $(2\alpha)^{-1}$ is the price decrease required to increase the provider market share by a factor of $\mathrm{e}^{1/2} \cong 1.65$ (i.e 65%).

If the prices are lowered, more customers will be attracted and E_i may eventually become larger or equal to one, $E_i \geq 1$. This corresponds to a case where all potential customers adopt the service and the market becomes *saturated* as shown in Fig. 1. The sum of n_{ij} must equal N_i, i.e.

$$\sum_j n_{ij} = N_i \tag{16}$$

This implies that:

$$n_{ij} = \frac{e_{ij}}{\sum_k e_{ik}} N_i = \frac{e_{ij}}{E_i} N_i \tag{17}$$

Note that the ratio of customers attracted by any two providers deciding to invest is again given by (13). To summarize:

- if $E_i < 1$ we have an unsaturated market where (11) holds and there are potential subscribers not adopting the service,
- if $E_i \geq 1$, we have a saturated market where all customers have adopted the service and n_{ij} is determined by (17).

The number of attracted customers are therefore written using a two-branch function:

$$n_{ij} = \begin{cases} N_i e_{ij} & \text{,if } E_i < 1 \text{ (unsaturated)} \\ \frac{1}{E_i} N_i e_{ij} & \text{,if } E_i \geq 1 \text{ (saturated)} \end{cases} \tag{18}$$

To gain further insight in this demand model, we show the market share, i.e. the fraction of customers attracted $n'_{ij} = n_{ij}/N_i$ from two providers $j = 1, 2$ in Figs. 2a and 2b respectively, assuming that $\alpha = 1$. We also show the penetration $n'_{\text{tot}} = n'_{i1} + n'_{i2}$ in Fig. 2c. In the latter figure, we observe the two market regimes: At high prices, the market is unsaturated and providers quickly attract more customers by lowering their prices. At some point, the market becomes saturated as n'_{tot} plateaus to 1. In this range, the competition between providers becomes more intense as the entire customer base is now covered and n'_{ij} depends on the fees charged by both competitors. We note that the distributions shown in Figs 2a and 2b exhibit discontinuous derivatives at the branch points of (18), which combined with the discrete values b_{ij} will complicate analytical equilibrium treatments.

3.3.2 Enhanced interaction model

We now consider an *enhanced interaction model* where subscribers in an area i do a good market search to choose the provider they will adopt. The best

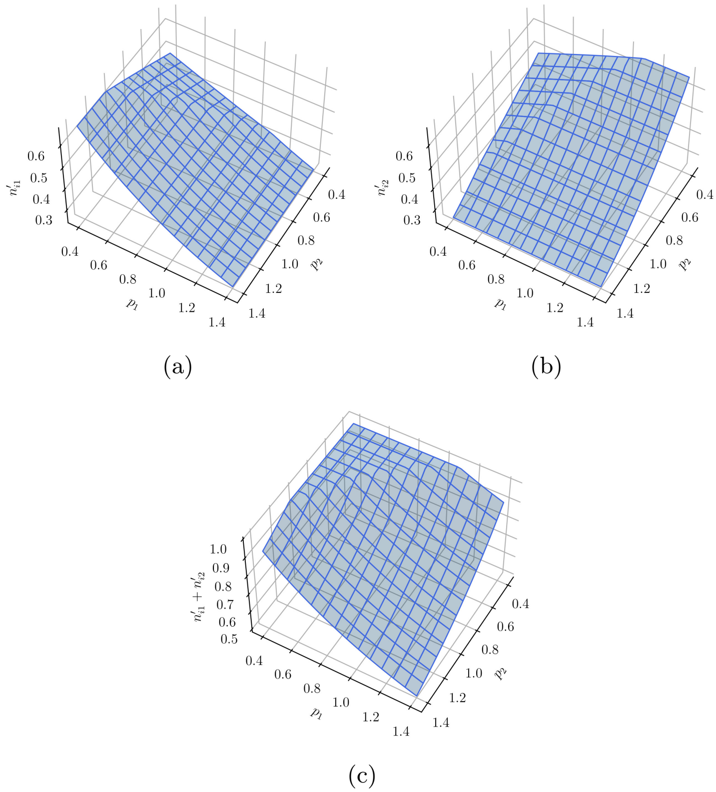

Fig. 2: Numerical examples of the attraction model described in Section 3.3.1, in the case of two competing providers deciding to invest in an area. Subfigures (a) and (b) show the market share of each provider as a function of the service fees. Subfigure (c) shows the total service penetration in the area.

option will be the provider $j_{\min}^{(i)}$ with the lowest price $p_{\min}^{(i)}$ given by:

$$p_{\min}^{(i)} = \min_j \{p_j \mid \text{ provided that } b_{ij} = 1\} \tag{19}$$

Assuming an exponential law as in (11), the total number of attracted subscribers is given by:

$$n_{\text{tot}}^{(i)} = N_i \mathrm{e}^{-\alpha p_{\min}^{(i)}} \tag{20}$$

For this model it is easy to show that α can be interpreted as the relative rate of change of the total number of attracted subscribers with minimum price,

$$r_{\text{tot}} \stackrel{\text{def}}{=} \frac{1}{n_{\text{tot}}^{(i)}} \frac{\partial n_{\text{tot}}^{(i)}}{\partial p_{\min}^{(i)}} = \alpha \tag{21}$$

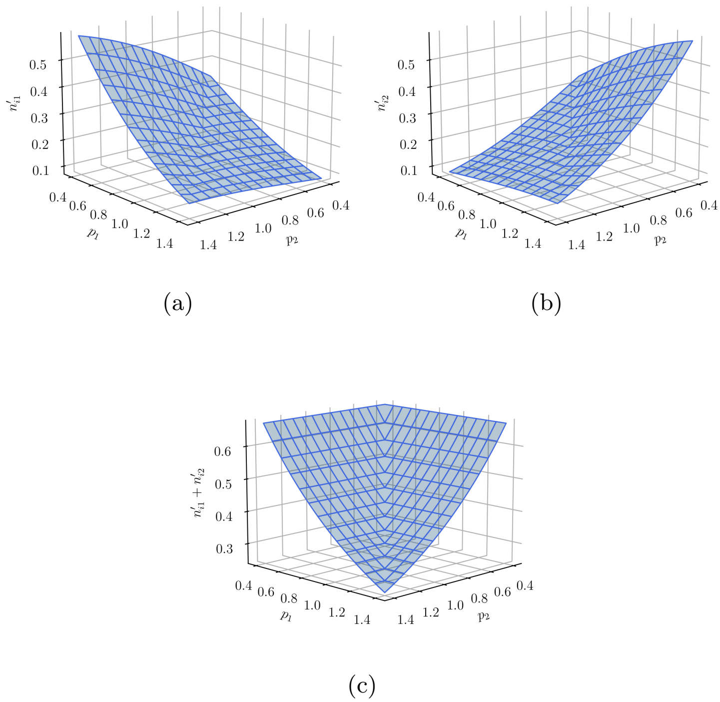

Fig. 3: Numerical examples of the attraction model described in Section 3.3.2, in the case of two competing providers deciding to invest in an area. Subfigures (a) and (b) show the market share of each provider as a function of the service fees. Subfigure (c) shows the total service penetration in the area.

We elaborate further by assuming that there is only one provider $j^{(i)}_{\text{min}}$ offering the cheaper rate, i.e. $p^{(i)}_{\text{min}} < p_j$ for all $j \neq j^{(i)}_{\text{min}}$. We expect that some a portion π_{ij} of subscribers will leak away from $j^{(i)}_{\text{min}}$ to another provider j that decides to expand in i, depending on the price difference:

$$\Delta p_{ij} = p_j - p^{(i)}_{\text{min}} \tag{22}$$

We again assume an exponential relation for the portion size:

$$\pi_{ij} \propto b_{ij} \mathrm{e}^{-\beta \Delta p_{ij}} \tag{23}$$

where β is some constant describing customer leakage. A small value of β suggests that subscribers may leak more easily to providers charging fees higher than $p^{(i)}_{\text{min}}$ since $\mathrm{e}^{-\beta \Delta p_{ij}}$ is slowly decreasing with increasing p_j. A large value of β implies that $\mathrm{e}^{-\beta \Delta p_{ij}}$ is rapidly decreasing with increasing p_j, implying smaller customer leakage. Since the portions must sum up to unity with respect to j, we can normalize π_{ij} as follows:

$$\pi_{ij} = \frac{b_{ij}\mathrm{e}^{-\beta\Delta p_{ij}}}{\sum_k b_{ik}\mathrm{e}^{-\beta\Delta p_{ik}}} \tag{24}$$

The number of subscribers is therefore:

$$n_{ij} = n_{\mathrm{tot}}^{(i)}\pi_{ij} \tag{25}$$

Note that (24) provides a further insight on the parameter β. Let us assume that only provider $j_{\mathrm{min}}^{(i)}$ offers the minimum price $p_{\mathrm{min}}^{(i)}$ while the rest offer fees p_j far greater, i.e. $p_j \gg p_{\mathrm{min}}$ for $j \neq j_{\mathrm{min}}^{(i)}$. In this case $\pi_{ij} \ll 1$ and from (24), we can show that[1]:

$$\frac{1}{\pi_{ij}}\frac{\partial \pi_{ij}}{\partial p_j} \cong -\beta \text{ ,when } j \neq j_{\mathrm{min}} \text{ and } p_j \gg p_{\mathrm{min}}^{(i)} \tag{26}$$

Therefore if one provider offers a much cheaper service, the relative decrease of the fraction π_{ij} of attracted customers by provider j is equal to β.

In case two (or more) providers $j = j_1, j_2, \cdots$, offer the minimum price, $p_{ij_1} = p_{ij_2} = \cdots p_{\mathrm{min}}^{(i)}$, then (24) suggests that they will have the same market share at the area in question i,

$$n_{ij_1} = n_{ij_2} = \cdots \tag{27}$$

Fig. 3a and Fig. 3b illustrate the number of subscribers attracted by two providers assuming $\alpha = 1$ and $\beta = 2$ while Fig 3c shows the service penetration. The latter is consistent with the fact that the minimum of p_1 and p_2 determines the subscribers opting to acquire the FTTH service by either provider. As shown in Figs 3a and 3b, the way in which subscribers are attracted by each provider changes around the diagonal $p_1 = p_2$. As long as $p_1 < p_2$ the market favors provider 1 while provider 2 is penalized by potential clients since dropping his fee brings less customers than when $p_2 < p_1$. We believe this is a better way to capture the market dynamics when well-informed clientele is assumed.

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[1]The customer fraction π_{ij} is written in terms of $f_j = A_j\mathrm{e}^{-\beta p_j}$ with $A_{ij} = b_{ij}\mathrm{e}^{\beta p_{\mathrm{min}}^{(i)}}$, as follows:

$$\pi_{ij} = \frac{f_j}{\sum_k f_k}$$

and if $\pi_{ij} \ll 1$:

$$\frac{\partial \pi_{ij}}{\partial p_j} = \frac{-\beta f_j}{\sum_k f_k} + \frac{\beta f_j^2}{\left(\sum_k f_k\right)^2} = -\beta(\pi_{ij} - \pi_{ij}^2) \cong -\beta\pi_{ij}$$

3.3.3 Incorporation of network characteristics

In this work, we assume that demand is predominantly shaped by pricing considerations, with network attributes such as throughput and latency not explicitly incorporated into our analysis. In FTTH, stability of service performance is inherent once the fiber connection is established at the customer's premises. This stability stands in contrast to legacy copper-based access technologies, where variables like distance from the cabinet significantly influenced service quality. In any case, if we wish to integrate network-specific attributes into utility functions, a robust approach involves employing multi-criteria methodologies such as the AHP [20]. This framework provides a systematic means of assessing the importance of individual parameters, enabling a more comprehensive analysis of network characteristics within the broader context of demand and pricing dynamics. By incorporating AHP, we can assess other wireless/wire line technologies, acknowledging the interplay between network attributes and consumer demand.

3.4 Parameter extraction

Before closing this section, we should note that the utility functions in (8), involve both deployment cost-related parameters (c_{ij} and d_{ij}) and demand-related parameters (α and β). The former are roughly estimated by the provider's engineers given a detailed map of the area and the house-hold geographical distribution, resulting in a fiber cable roll-out plan. A detailed calculation of deployment-related costs (trenching, road restoration, fiber termination, etc) as in [24], can be used to obtain an estimate of the values of c_{ij} and d_{ij}. Estimation of the demand-related parameters is more involved. One approach is to fit the demand parameters with actual customer data [41]). The detailed manner in which the demand and cost-related parameters are extracted is beyond the scope of this paper. That being said, diagrams of Figs. 2 and 3 provide some intuition on the value range of α and β, considered in the simulations. Alternatively, we could resort to time-series forecasting adopted in [34] based on earlier access technology generations, nonlinear Lotka–Volterra equations [42], multi-generational models [43] or fitting demand parameters with actual data [41]. These alternatives can be incorporated in our implementation of the Nash GA described in Section 4.

4 Solution Engine

The solution search space contains both the flat prices p_j of each provider which are continuous variable and the decision variables b_{ij} which are binary. This constitutes a mixed-variable problem and in light of the complexity of the utility functions containing branches as in (18) or minimization in (20), analytical treatment becomes intractable. We therefore need to resort to numerical frameworks such as the Nash GA outlined in this section. An overview of the Nash GA applied in this context is depicted in Figure 4. The details are discussed below.

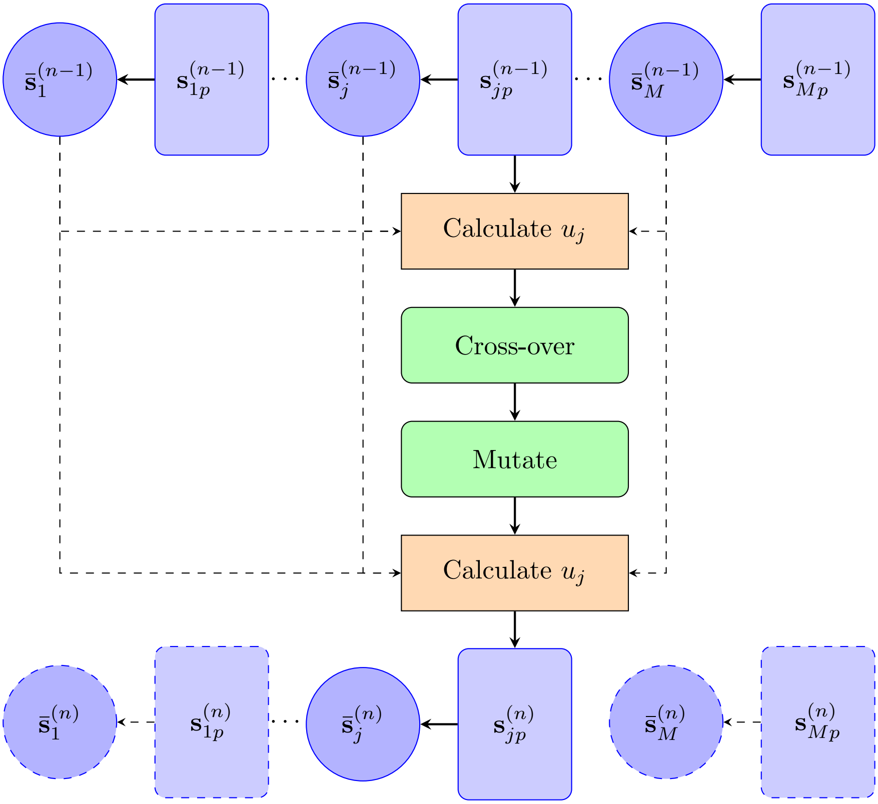


Fig. 4: Overview of the Nash GA.

4.1 Overview of the algorithm

In this section, we describe the details of the genetic algorithm used to solve (5) and find the optimal action profiles $\mathbf{s}^*$ [10]. For each player j, we maintain a pool of $\mathbf{P}_j^{(n)} = [\mathbf{s}_{jp}^{(n)}]$ of P possible action plans $\mathbf{s}_{jp}^{(n)}$ where $1 \leq p \leq P$. The pool is updated in each iteration n, according to Figure 4. Let $\bar{\mathbf{s}}_j^{(n-1)}$ be the best action profile of player j obtained in the previous iteration $n-1$ (we shall discuss how this action plan is chosen later on). At the beginning of each iteration, n, we take as a starting point the pools obtained in the previous iteration $\mathbf{P}_j^{(n-1)}$, i.e. we set $\mathbf{s}_{jp}^{(n)} \leftarrow \mathbf{s}_{jp}^{(n-1)}$.

For each player j, we first calculate the utilities u_{jp} for the action plans in his pool $\mathbf{P}_j$ assuming that the other players k $(k \neq j)$ conform to the best action plan obtained at the previous iteration, $\bar{\mathbf{s}}_k^{(n-1)}$. Following the notation introduced in (3), this is formally written as:

$$u_{jp} = u_j\left(\bar{\mathbf{s}}_{-j}^{(n-1)}, \mathbf{s}_{jp}^{(n)}\right) \tag{28}$$

where $(\bar{\mathbf{s}}_{-j}^{(n-1)}, \mathbf{s}_{jp}^{(n)})$ is the action profile obtain when player j chooses action $\mathbf{s}_{jp}^{(n)}$ from its pool while the rest of the players conform to their optimal action plans obtained from the previous iteration. The next step involves the *cross-over* operation where members of the pool $\mathbf{P}_j$ are combined to produce *offsprings*, i.e. new candidate action plans. In order to maintain population diversity, we also *mutate* the offsprings, i.e. we randomly perturb their variable values. The utility functions for these new action plans are calculated again using (28). If we obtain a higher utility function than those already found in the pool, then we replace the weakest pool members by the superior action plans. This provides the new population pool $\mathbf{P}_j^{(n)}$ in which we identify the action $\bar{\mathbf{s}}_j^{(n)}$ plan with the for player j with the highest utility value. This procedure is carried out for all players in order to obtain the new populations pools and the new optimal action plans to feed the next iteration $n+1$ of the algorithm. After a number of iterations, the algorithm will converge to a Nash equilibrium, [10], [44]. In the following subsections we discuss several implementation aspects of the algorithm.

4.2 Variables

As shown in (1), the action plans $\mathbf{s}_j$ consist of continuous variables p_j and binary variables b_{ij}, indicating a *mixed-variable* optimization problem. We modify the Nash GA proposed in [10] to adopt the mapping of conventional GAs for mixed problems [35]. Let g_{kj} be real variables bound inside $[0,1]$ for $1 \leq k \leq N+1$ and $1 \leq j \leq M$. The continuous variables p_j are mapped as follows:

$$p_j = g_{N+1,j}(P_{\max} - P_{\min}) + P_{\min} \tag{29}$$

where $P_{\max}$ and $P_{\min}$ are the assumed upper and lower bounds for the prices p_j, $P_{\min} \leq p_j \leq P_{\max}$. We can also consider different bounds for each player if the demand-model parameters are quite different. We can choose $P_{\min} = 0$ and set $P_{\max}$ to a value high enough, that the number of subscribers in the absence of any competition is very low. For the binary variables b_{ij} we use the mapping,

$$b_{ij} = [g_{ij}] \tag{30}$$

where $1 \leq i \leq N$ and $[x]$ is the integer closest to x. Equations (29) and (30) imply that instead of using both real and binary variables, we can simply store the real parameters $g_{kj}^{(p)}$ bound inside $[0,1]$, where p denotes the index of the pool member in the population pools and therefore treat all variables in a unified manner.

4.3 Cross-over and mutation

In each iteration, the strongest 50% of the population members constitute the mating pool [35]. We choose two parents through tournament selection and calculate the offspring using uniform crossover, which consists of tossing an unbiased coin and randomly selecting the value of each offspring variable from either the first or the second parent. This produces an offspring with parameters equal to $\bar{g}_{kj}^{(p)}$ which are a mix of the parameters of the parents. Mutation is achieved by adding a random perturbation $\Delta g_{kj}^{(p)}$ to each of the parameters determined by:

$$\Delta g_{kj}^{(p)} = m_0 r_{kj}^{(p)} g_{kj}^{(p)} \tag{31}$$

where $0 \leq m_0 \leq 1$ is the *mutation factor* and $r_{kj}^{(p)}$ are randomly chosen inside $[-1, 1]$ from a uniform distribution. The values of the parameters after the mutation $\tilde{g}_{kj}^{(p)}$, are given by the fractional parts of the perturbed variables,

$$\tilde{g}_{kj}^{(p)} = \bar{g}_{kj}^{(p)} + \Delta g_{kj}^{(p)} - \lfloor \bar{g}_{kj}^{(p)} + \Delta g_{kj}^{(p)} \rfloor \tag{32}$$

where $\lfloor x \rfloor$ denotes the integer part of x.

4.4 Initialization and termination

The initialization of the initial pools is carried out by randomly choosing the initial values of $g_{kj}^{(p)}$. These values yield the starting action plans $\mathbf{s}_{jp}^{(0)}$ for each player through the transformations discussed in Section 4.2. We can randomly choose one of these action plans in each pool to be the starting optimal action plan $\bar{\mathbf{s}}_j^{(0)}$. The criteria for terminating the algorithm may vary depending on the requirements. In our implementation, we evaluate the degree of convergence by measuring the relative variation δu_j of the utility functions u_{jp} for each player pool,

$$\delta u_j = 1 - \frac{\min_p \{u_{jp}\}}{\max_p \{u_{jp}\}} \tag{33}$$

We assume that convergence is achieved if δu_j is smaller than a specified value $\delta u_{\min}$ for all player pools $\mathbf{P}_j$.

If a complex demand model is adopted, the utility function evaluation is expected to be the predominant factor in the execution time of the algorithm. We may therefore choose to terminate the algorithm after the number of total utility function evaluations exceed a specified number N_{evals}.

4.5 Implementation

Our `Python` implementation is publicly available under an open-source license [38]. It relies on the `NumPy` module which is a fundamental scientific computing package. We also make use of the `Matplotlib` package for visualization of the results. In our implementation, we adopted an object-oriented approach where the various data structures involved in the algorithm are represented as `classes`. The variables are represented by the `chromosome` class which provides the necessary methods for manipulating the parameters $g_{kj}^{(p)}$ and their mapping to the actual action variables $p_j^{(p)}$ and $b_{ij}^{(p)}$ for each member p of a population pool for player j. The `population` class, deals with the pool of action plans for each player providing the necessary tools for calculating the utility of each action plan, carrying out the cross-over and mutation operations and sorting the action plans in terms of fitness. The ensemble of pools is described by the `population_group` class, which handles the calculation of the utility functions at a group level (i.e. taking into account the best action plans of the other players), the calculation of the next generation of pools and keeping track of the overall convergence of the algorithm in terms of the value range of utility values for each pool and provides some rudimentary logging. The `operator_game` class models the access provider competition incorporating the two demand models discussed in Section 3.

5 Results and discussion

We first discuss single area games where the provider interaction is more easily understood and then move on to multiple area games.

5.1 Single area games

Table 1: Game parameters

Parameter	Explanation	Value
N_1	Max. number of potential customers	10.000.000
M	Number of players	2
N	Number of areas	1
c_{1j}	Average connection cost (per subscriber)	0.2
α	Demand coefficient	1
β	Leakage coefficient	2
P	Pool size per player	10
m_0	Mutation factor	0.1
$\delta u_{\min}$	Minimum required pool utility range	10^{-3}
N_{evals}	Maximum number of utility evaluations	10^4

We begin by examining a single area game, $N = 1$ with two players, $M = 2$, reflecting a situation where the operators decide whether or not to offer the service at a national scale simultaneously. Table 1 summarizes the key parameters assumed. We have chosen the maximum customer number size N_1 that from a practical stand-point could make sense to invest on the entire region directly and not on an area-by-area basis. The demand coefficient α is equal to 1 as in Figs 2 and 3. To negate the need to transform prices between currencies, we have adopted an arbitrary currency unit for p_j which varies in $[0, 4]$. Given the $\mathrm{e}^{-\alpha p_j}$ dependence in (10), the range $0 \leq p_j \leq 4$ corresponds to service penetration ranges from 2% to 100%. For the enhanced interaction model, we set the leakage coefficient in (23) equal to $\beta = 2$. We will investigate the influence of this parameter later on in this section. We also assume an average connection cost equal to $c_{1j} = 0.2$ per subscriber.

Table 2: Nash equilibrium for $N = 1$ and the limited interaction model

Parameter	Explanation	Optimal value
u_1	Utility (profit) for player 1	3.01×10^6
u_2	Utility (profit) for player 2	3.01×10^6
p_1	Price set by player 1	1.20
p_2	Price set by player 2	1.20
b_{11}	Player 1 decision variable	1 (True)
b_{12}	Player 2 decision variable	1 (True)
n_1	Total subscribers for player 1	3.0×10^6
n_2	Total subscribers for player 2	3.0×10^6

5.1.1 Limited interaction model

Table 2 summarizes the Nash equilibrium point assuming the limited interaction model. We readily see that the optimal action plans for each player are identical, due to the nature of the limited interaction model. Given that $n_1 + n_2 < N_1$, the market is unsaturated and each provider attracts customers independently of the other. It is in the best interest of each provider to offer the service ($b_{1j} = 1$) at a price $p_1 = p_2 \cong 1.2$ much higher than the connection cost ($= 0.2$). Figure 5a shows the obtained utilities u_j corresponding to the best action plans of each player j, with respect to the iteration number n which is an indication of the convergence of the algorithm. The algorithm terminates after 162 iterations and 3584 utility function evaluations, achieving the required relative utility variation.

Figures 5b and 5c show the evolution of the corresponding subscriber numbers n_j and the prices p_j. We see that a similar convergence behavior is obtained, since the values of these parameters do not significantly vary after 70 iterations. Figure 5d shows a sensitivity analysis around the equilibrium values of the prices p_1 and p_2 of Table 2. This is carried out by calculating the utility functions

$$u_1' = u_1(p_1 + \Delta p, b_{11}, p_2, b_{12}) \tag{34}$$

$$u_2' = u_2(p_1, b_{11}, p_2 + \Delta p, b_{12}) \tag{35}$$

As shown in Figure 5d, $\Delta p = 0$ is a maximum for both u_1' and u_2', implying that neither player can do better by changing the price. Additionally, we see that starting from the equilibrium point and choosing $b_{11} = 0$ would imply that the utility for player 1 will drop to zero. The same holds for player 2 when

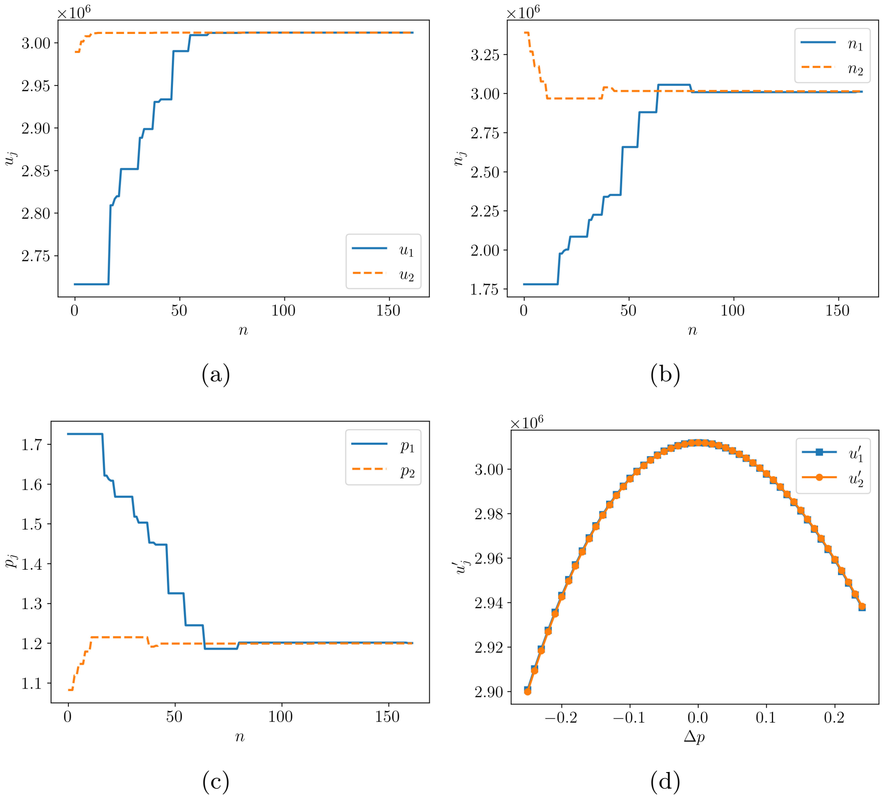


Fig. 5: Results of the Nash GA for the case of the limited interaction model assuming all players have the same installation costs: a) Utilities (profits) u_j, b) number of subscribers n_j, c) prices p_j as a function of the algorithm iteration n, d) sensitivity analysis around the equilibrium point.

setting $b_{12} = 0$. Thus neither provider can do better by changing their action plans and Table 2 indeed provides a Nash equilibrium.

5.1.2 Enhanced interaction model

We next apply the algorithm to find the equilibrium for the enhanced interaction model. Table 3 shows the equilibrium point obtained by the algorithm. Unlike Table 2, player 2 now has a higher utility function u_2 and achieves this by offering the service at a substantially lower price p_2 which allows him to establish a leading position in the market, attracting a much higher subscriber number n_2 than his competitor, player 1. On the other hand, player 1 seeks to strike a balance between lowering his rate p_1 in order to attract more customers and keeping the price high enough to ensure significant revenues. Re-initializing at different initial random states, we confirmed that there are two equilibrium points in which the role of player 1 and 2 are exchanged. Under

Table 3: Nash equilibrium for $N = 1$ and the enhanced interaction model

Parameter	Explanation	Optimal value
u_1	Utility (profit) for player 1	1.43×10^6
u_2	Utility (profit) for player 2	1.63×10^6
p_1	Price set by player 1	1.01
p_2	Price set by player 2	0.77
b_{11}	Player 1 decision variable	1 (True)
b_{12}	Player 2 decision variable	1 (True)
n_1	Total subscribers for player 1	1.76×10^6
n_2	Total subscribers for player 2	2.87×10^6

this interaction model, providers may not prefer to simply lower the price to cope with the competition. A sensitivity analysis around the equilibrium point of Table 3 is carried out in Figure 6 which shows of u'_1 and u'_2 in (34) and (35). Since $\Delta p = 0$ is a maximum point for both utilities, we see that neither player alone can choose a better price strategy. We notice a bump-like behavior near $\Delta p = -0.24$ for u'_1 and $\Delta p = +0.24$ for u'_2. At the former point, player 1 sets a price $p_1 = 0.77 = p_2$ and hence the providers attract the same portion of subscribers from the customer pool. This turns out to be sub-optimal due to the enhanced interaction assumed. Similarly, at $\Delta p = 0.24$, player 2 sets the same price as 1 ($p_2 = p_1 = 1.01$) and this turns out to be a sub-optimal strategy for player 2.

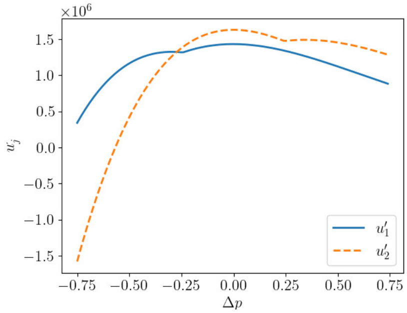


Fig. 6: Price sensitivity analysis around the equilibrium point of Table 3.

Figure 7 shows the convergence properties of the algorithm for the enhanced interaction demand model. Figure 7a shows the evolution of utilities u_j corresponding to the best action plans of each player. Figures 7b and 7c show the evolution of the corresponding subscriber numbers n_j and the prices p_j. In this case the algorithm terminates after 179 iterations and 3598 utility function calls, achieving the desired utility value range in both population pools.

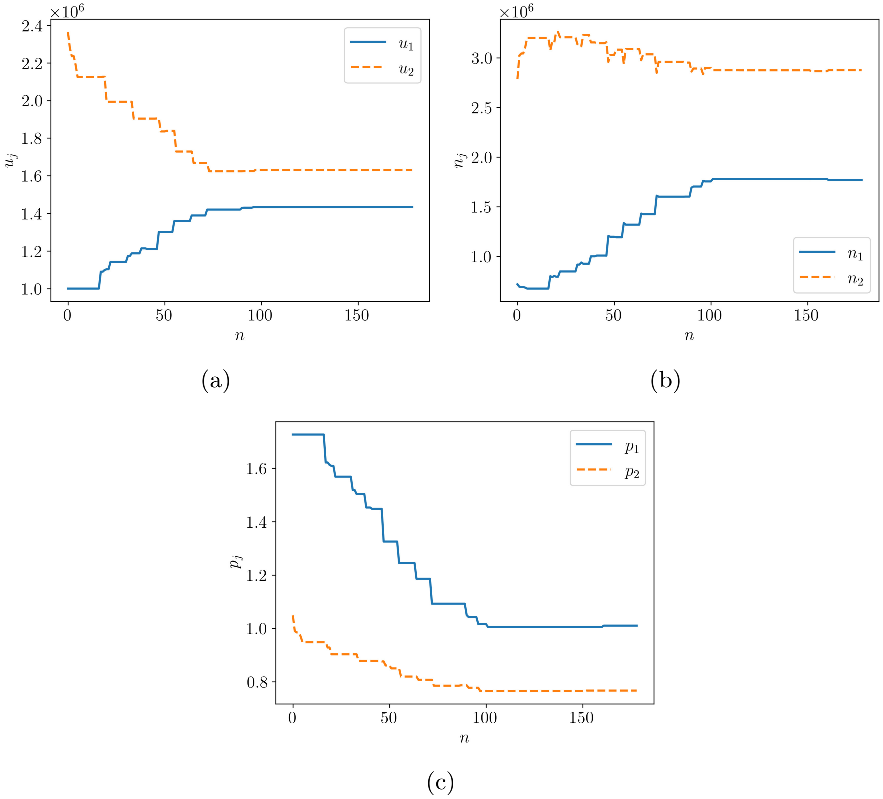


Fig. 7: Results of the Nash GA for the case of the enhanced interaction model assuming all players have the same installation costs: a) Utilities (profits) u_j, b) number of subscribers n_j, c) prices p_j as a function of the algorithm iteration n.

5.1.3 Influence of customer awareness

The effect of β, describing customer leakage in the enhanced interaction case, is shown in Figure 8. Figure 8a shows the equilibrium utility values obtained for various values of β in the range $[2, 5]$ while Figures 8b and 8c show the corresponding subscriber populations and optimal price settings. In Section 3.3.2, we discussed how β can be related to consumer awareness. According to (23), a high value for β implies that few customers will be attracted by the expensive provider. This pushes the players to lower their prices p_j (Figure

8c), thereby attracting higher subscriber numbers n_j (Figure 8b). The provider profits are however reduced as shown in Figure 8a.

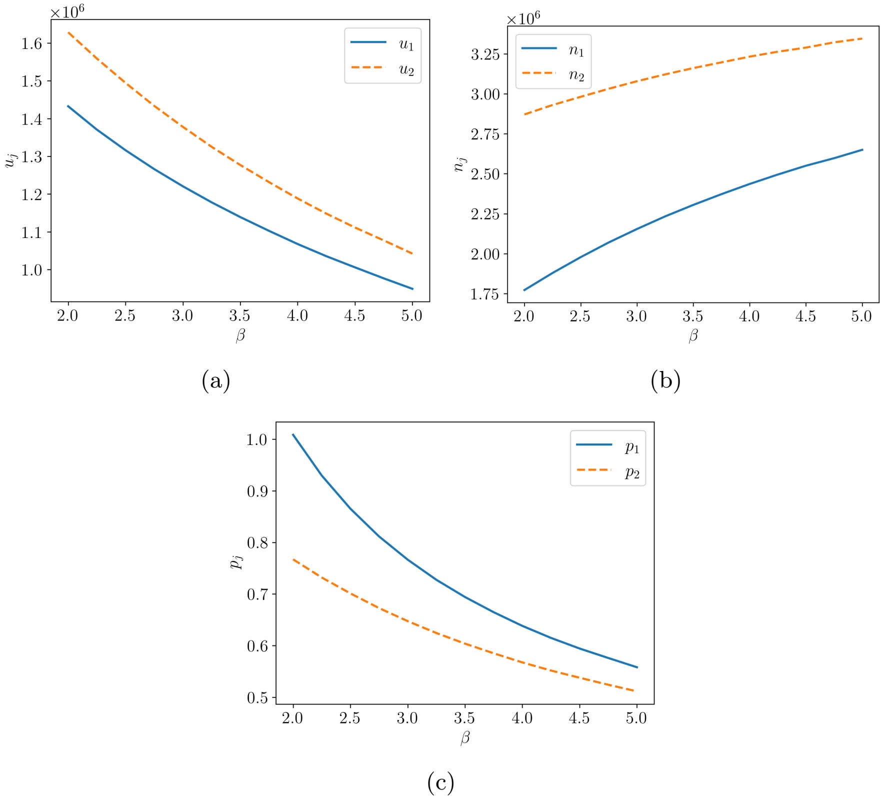


Fig. 8: Nash equilibrium obtained for different β: a) Utilities (profits) u_j, b) number of subscribers n_j, c) prices p_j.

5.1.4 Dissimilar providers

We next take a look at a case where installation costs for players are dissimilar. Table 4 describes the equilibrium point obtained when the average connection costs are $c_{11} = 0.2$ (same as before) and $c_{12} = 0.5$. Player 2 must now compromise with a lower utility value u_2 due to the reduced profit margin. Player 1's utility remains unchanged compared to Table 2. As explained in Section 3.3.1, under the limited interaction model, the players attract customers independently unless the market is saturated. Table 5 outlines Nash's equilibrium for dissimilar providers, in the case of enhanced interaction. Since player 2 has a reduced profit margin, player 1 now becomes dominant with a utility value which is higher than the utility of the dominant player in Table

3. Player 2 is forced to charge higher rates, thereby claiming smaller portion of the customers willing to pay for the service.

Table 4: Nash equilibrium for $N = 1$ with dissimilar providers under the limited interaction model

Parameter	Explanation	Optimal value
u_1	Utility (profit) for player 1	3.01×10^6
u_2	Utility (profit) for player 2	2.23×10^6
p_1	Price set by player 1	1.20
p_2	Price set by player 2	1.49
b_{11}	Player 1 decision variable	1 (True)
b_{12}	Player 2 decision variable	1 (True)
n_1	Total subscribers for player 1	3.0×10^6
n_2	Total subscribers for player 2	2.2×10^6

Table 5: Nash equilibrium for $N = 1$ with dissimilar providers under the enhanced interaction model

Parameter	Explanation	Optimal value
u_1	Utility (profit) for player 1	1.88×10^6
u_2	Utility (profit) for player 2	0.98×10^6
p_1	Price set by player 1	0.81
p_2	Price set by player 2	1.22
b_{11}	Player 1 decision variable	1 (True)
b_{12}	Player 2 decision variable	1 (True)
n_1	Total subscribers for player 1	3.05×10^6
n_2	Total subscribers for player 2	1.35×10^6

5.2 Multiple area games

5.2.1 Nash equilibrium

We next discuss games where each player adjusts the variables β_{ij} of his action plan in multiple areas ($N > 1$). This scenario is more interesting from a practical point-of-view, since access providers usually take decisions on a area-by-area basis. Such scenarios are harder to solve because of the much higher dimension in the variable search space. We consider a ten area game ($N = 10$) in which have different average customer connection cost c_{ij} which is the same for all providers ($c_{ij} = c_i$). Table 6 summarizes the game parameters assumed.

Table 6: Multiple area game parameters

Parameter	Explanation	Value
N_i	Area population	1.000.000
M	Number of players	2
N	Number of areas	10
c_{1j}	Average connection cost (per subscriber)	$0.3(i-1)+0.2$
α	Demand coefficient	1
β	Leakage coefficient	2
P	Pool size per player	50
m_0	Mutation factor	0.1
$\delta u_{\min}$	Minimum required pool utility range	10^{-3}
N_{evals}	Maximum number of utility evaluations	10^6

Seeking an analytical solution for this problem is even more involved than in single are games: one must consider many candidate sets of values for b_{ij} in addition to discontinuities in the derivatives of utility functions with respect to price.

Table 7: Nash equilibrium for $N = 10$ under the limited interaction model

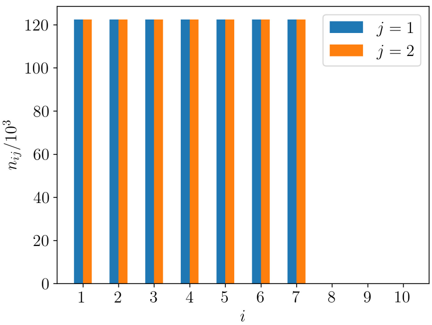
Parameter	Explanation	Optimal value
u_1	Utility (profit) for player 1	0.85×10^6
u_2	Utility (profit) for player 2	0.85×10^6
p_1	Price set by player 1	2.10
p_2	Price set by player 2	2.10
b_{i1}	Player 1 decision variable	1 for $i \leq 7$, 0 otherwise
b_{i2}	Player 2 decision variable	1 for $i \leq 7$, 0 otherwise
n_1	Total subscribers for player 1	0.85×10^6
n_2	Total subscribers for player 2	0.85×10^6

Due to the higher dimension of the search space, we need a larger pool size per player P to ensure population diversity, and we have chosen $P = 50$ in the simulations. The larger pool size leads to increased number of required utility function evaluations. For c_{ij}, we have assumed a simple linear dependence on i, $c_{ij} = 0.3(i-1) + 0.2$. Table 7 shows the equilibrium obtained based on the limited interaction model. The optimal action plans are the same for both players similar to the single area game (Table 2). The figure suggests that the providers prefer to invest in areas with lower installation costs c_{ij} first (smaller i). The higher connection costs in some areas decrease the profits and providers are better off not expanding there ($b_{i1} = b_{i2} = 0$ for $i \geq 8$).

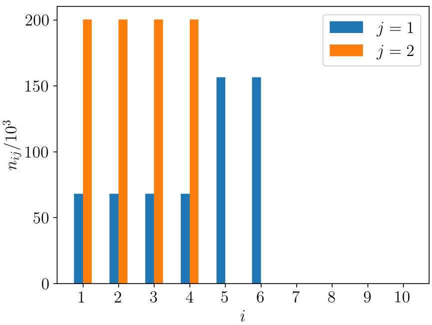
Table 8: Nash equilibrium for $N = 10$ under the enhanced interaction model

Parameter	Explanation	Optimal value
u_1	Utility (profit) for player 1	0.42×10^6
u_2	Utility (profit) for player 2	0.53×10^6
p_1	Price set by player 1	1.85
p_2	Price set by player 2	1.31
b_{i1}	Player 1 decision variable	1 for $i \leq 6$, 0 otherwise
b_{i2}	Player 2 decision variable	1 for $i \leq 4$, 0 otherwise
n_1	Total subscribers for player 1	0.58×10^6
n_2	Total subscribers for player 2	0.80×10^6

Table 8 summarizes the equilibrium point for the enhanced interaction model, where the role of the players is distinguished. Figure 9 illustrates the customer distribution n_{ij} attracted in each area i by provider j. In the limited interaction model, this distribution is identical for both providers but in the case of enhanced interaction, the distributions differ. The dominant player (player 2) is able to achieve a higher utility value by expanding in fewer areas ($b_{2i} = 0$, for $i \geq 4$). The alternative provider must choose to expand in less profitable areas ($b_{1i} = 0$ for $i \geq 6$) in order to make up some of the profit loss. These are examples of how our framework can assist providers in identifying their investment plans given each region characteristics.



(a)

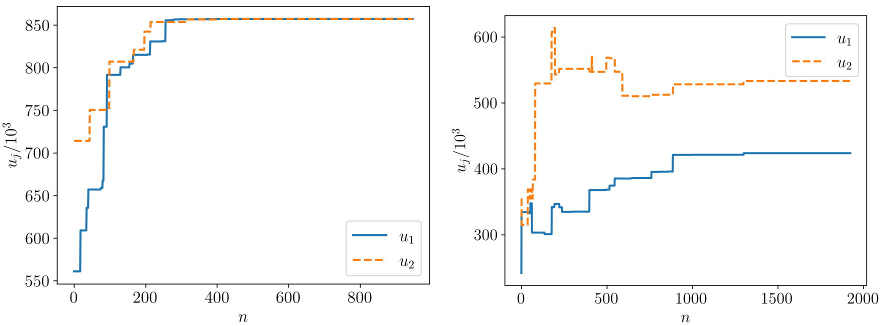


(b)

Fig. 9: Subscribers attracted in the 10-area game for (a) the limited and (b) the enhanced interaction models.

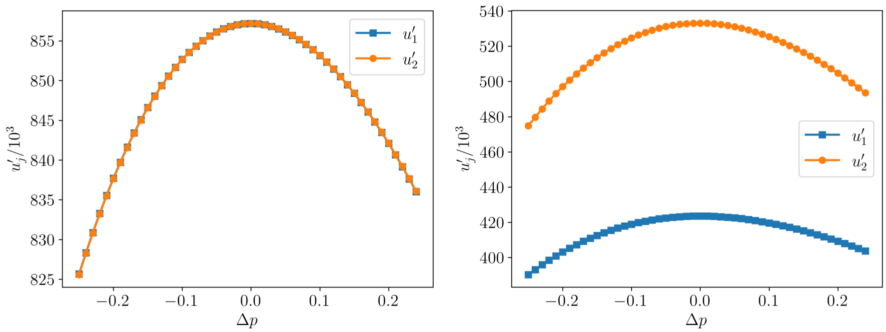
5.2.2 Convergence and sensitivity analysis

We next discuss the algorithm convergence in the multiple area scenarios. In Figure 10, we show the optimal utilities u_j for both players at each iteration of the algorithm. For brevity we omit the corresponding graphs for the subscriber number n_j and the prices p_j which more or less exhibit the same behavior. Looking at the horizontal axis scale, it is evident that convergence is slower than Figs. 5 and 7 due to the increase in search space dimension. The search space now comprises of the two continuous variables p_j plus 20 discrete binary variables b_{ij}, instead of 2 plus 2 in the single area game. Figure 10a corresponds to limited provider interaction model where the algorithm terminates after 948 iterations ($96,796$ utility function evaluations). Figure 10a shows the results of enhanced interaction where the algorithm terminates after $1,924$ iterations ($196,348$ utility function evaluations). In Figure 11 we show the sensitivity analysis with respect to p_j. We ascertain that $\Delta p = 0$ is maximum around the estimated equilibrium point. Figure 12 shows the results of a sensitivity analysis carried out by inverting just one decision variable of the players (i.e. setting $b_{ij} = 1$ if $b_{ij} = 0$ and vice-versa). We readily see that for both demand models, the change $\Delta u_j = u'_j - u_j$ is negative as expected in a Nash equilibrium. The sensitivities in the limited interaction game are identical, since the solution is completely symmetric. In the enhanced interaction model, the dominant player (player 2) has the highest sensitivity values as expected.



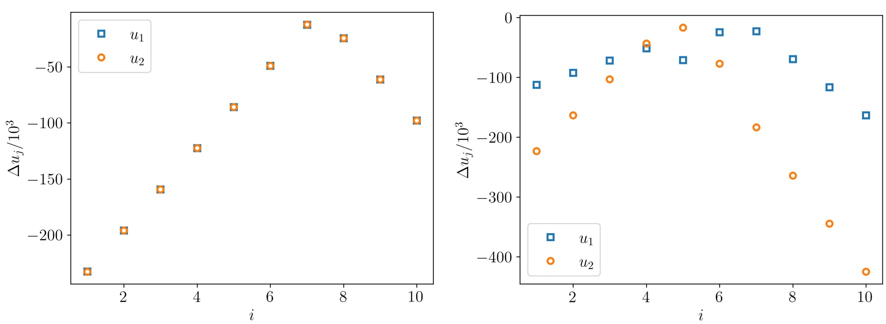
(a) (b)

Fig. 10: Optimal utilities for the two players as a function of the algorithm iteration n for the case of a) the limited and b) the enhanced interaction models respectively.



(a) (b)

Fig. 11: Price sensitivity analysis around the equilibrium point for the ten area game for the case of a) the limited and b) the enhanced interaction models.



(a) (b)

Fig. 12: Decision sensitivity analysis around the equilibrium point for the ten area game for the case of a) the limited and b) the enhanced interaction models.

6 Conclusions and Future Work

We have illustrated how a properly modified Nash GA can be used in order to calculate the equilibrium point in the case of access providers which compete over the provision of a given flat-rated broadband service such as FTTH. We considered two simple demand models. In the first, the competition is limited and each operator attracts subscribers independently of the other, unless the market becomes saturated. In the second model, the total number of subscribers are determined by the provider with the lowest price and other providers draw subscribers from it depending on the difference of their charged price compared to the minimum price. We have shown that the algorithm can handle cases where there are several areas in which each provider can decide whether he wishes to expand or not, depending on the area particularities. Our work can be used to implement decision support tools which can help providers to evaluate the market potential of a broadband technology and regulatory bodies to understand the competition prospects and protect the customer interests.

Our `Python`-based "solution engine" is publicly available on the web. Possible extensions would be to reduce the computation time by identifying the right heuristics for initializing the algorithm in order to speed up the convergence process, instead of using random initial states. Alternative evolutionary approaches such as colonial competitive algorithms proposed in [45] can also be studied to ascertain whether they can speed-up convergence. Apart from the engine itself it would be interesting to consider alternative demand models in the presence of competition. These include Markov chains, curve-fitting, etc. One should also adopt a more fine-grained cost model for estimating deployment costs detailed in [46]. In FTTH, such costs depend on pre-installed fiber routes (i.e. dark fibers), the position of the provider's point-of-presence (PoPs), population density, terrain type, etc [24]. One could also take into account various inter-relations between the territories, e.g. expanding to area B is easier when one is already present in area A. We can also consider scenarios where an incumbent player is already present in some areas therefore his decision variables may be locked. We plan to address some of these issues in future publications.

Conflict of interest. The authors have no competing interests to declare that are relevant to the content of this article.

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